Uncertainty in the measurement of low-frequency magnetic fields — Assessment of the perturbation due to the operator and to the exposed subject

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The standards regulating human exposure to electromagnetic fields require that the exposure levels to be compared with the limit values be determined under "unperturbed" conditions, which means:

- having removed from the exposure scenario the exposed subject whose exposure is to be quantified;
- having eliminated, or reduced as much as possible, the perturbation due to the possible presence of the operator performing the measurement.

The second of these conditions can be met fairly easily, by moving the operator away from the source and from the measuring instrument by means of (insulating) tripods and remote control devices.

The first condition, on the other hand, can be quite difficult to put into practice, especially in cases such as industrial arc welding or the use of electrosurgical units, where the activation of the source is strictly tied to the presence of the operator who uses it and even holds it in their hand. In the first of these cases (arc welding), however, it is often pointed out that the source mainly emits a magnetic field of (relatively) low frequency, which is not affected by the presence of the welding operator.

It is well known, in fact, that the low-frequency magnetic field does not interact significantly with the exposed conductive objects (as long as they are not ferromagnetic) and is therefore little or not at all perturbed by their presence in the exposure scenario. Such objects include human organisms as well, hence the bodies of the operator performing the measurement and of the exposed subject.

Strictly speaking, though, every time-varying magnetic field induces electric currents in the exposed conductive objects and is therefore perturbed, to some extent, by the magnetic field generated by those very currents. The perturbation is ascribable to the electromagnetic induction governed by Faraday's law, so its relevance is proportional to the frequency of the field, to the (average) conductivity of the exposed object and to the area of the object's cross-section orthogonal to the field direction (Figure 1). In this technical note we want to estimate under which conditions (and in particular up to which frequency value) this perturbation is negligible within a preset accuracy level.

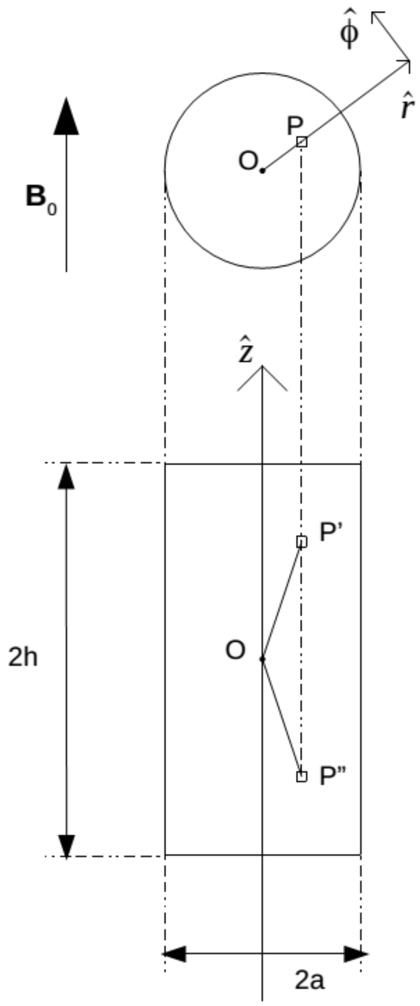

Figure 1. *Model of the exposed object: a cylinder of diameter 2a and height 2h, with its axis parallel to the primary magnetic field B_0. P′ and P″ are two volume elements placed symmetrically with respect to the mid-plane.*

To identify these conditions, let us consider a particular case which, although extremely simple and schematic, is nevertheless able to provide useful indications. With reference to Figure 1, we model the exposed object by a cylinder made of homogeneous material with electrical conductivity σ, diameter 2a, height 2h and longitudinal axis parallel to the direction of the "primary" magnetic field of flux density B_0. Let f denote the frequency of the field, and let us refer to a cylindrical coordinate system (r, φ, z) with the z axis coinciding with the direction of the primary field and the origin at the geometrical centre O of the cylinder. In this configuration, the expression of the current density J(P) induced at a generic point P(r, φ, z) of the object is known from theory:

$$J\left(P\right) = -j\sigma f\pi r B_0 \varphi$$

(directed along the azimuthal unit vector) (Eq. 1)

Starting from this expression it is possible to compute (by means of the first Laplace formula, i.e. the Biot–Savart law) the contribution $dB'_1(O)$ to the secondary field at the centre O (chosen, for simplicity, as the evaluation point) due to the current density J(P′) present in the volume element dV around the generic point P′ of the cylinder, and the contribution $dB''_1(O)$ due to the current density J(P″) present at the point P″ located symmetrically to P′ with respect to the mid-plane of the cylinder. One finds:

$$\begin{aligned} dB'_1(O) &= \mu_0 \frac{J(P')\,dV \times (O-P')}{4\pi\,|O-P'|^3} \\ dB''_1(O) &= \mu_0 \frac{J(P'')\,dV \times (O-P'')}{4\pi\,|O-P''|^3} \end{aligned} \qquad \text{(Eq. 2)}$$

If we use in these the expression of Eq. 1 for the current density J, write out the vector coordinates in the chosen cylindrical system and sum the two symmetrically located contributions, we obtain as a result a vector dB_1 aligned with the z axis of the cylinder (and hence with the primary magnetic field B_0), whose magnitude is:

$$\mathrm{d}B_1\,(O) = \frac{\mu_0 \sigma f B_0 r^3}{2\sqrt{\left(r^2+z^2\right)^3}} \mathrm{d}r d\varphi \mathrm{d}z \qquad \text{(Eq. 3)}$$

Finally, integrating this expression over the whole volume of the cylinder, one determines the expression $B_1(O)$ of the magnitude of the magnetic flux density produced by the induced currents:

$$B_1\,(O) = \mu_0 \sigma f \pi B_0 h \left(\sqrt{a^2+h^2} - h\right) \qquad \text{(Eq. 4)}$$

A handier expression, but equally suited to our purpose, can be obtained by considering an infinitely long cylinder, i.e. by taking the limit for $h \longrightarrow \infty$ (see the Appendix); one finds:

$$B_1\,(O) = \frac{1}{2} \mu_0 \sigma f \pi B_0 a^2 \qquad \text{(Eq. 5)}$$

From this expression it is finally possible to derive the condition that must be satisfied by the parameters of the problem (frequency f, radius a of the exposed cylinder and conductivity σ of its constituent material) if the ratio B_1/B_0 (which quantifies the extent of the perturbation of the magnetic field caused by the induced currents) is to remain below a preset limit ε. The resulting expression is the following:

$$\sigma f a^2 < \frac{2\varepsilon}{\pi \mu_0} \qquad \text{(Eq. 6)}$$

Having referred, for ease of computation, to the geometrical centre O of the exposed conductive object is not an excessive limitation, because at point O the contribution of the field produced by the induced currents is at its maximum, and therefore the evaluation performed is useful to estimate the maximum expected value of the perturbation of the primary field caused by the exposed object.

As an example, if — thinking of an exposed individual — we take the values $\sigma \approx 0.3$ S/m (typical value of human muscle tissue at low frequency) and $a \approx 30$ cm, the perturbation of the magnetic field turns out to be smaller than 1% for all frequencies up to beyond 180 kHz, and smaller than 3 ppm at 50 Hz.

A somewhat more detailed analysis is provided by the graph in Figure 2. It shows several curves, corresponding to different values of the parameter a (the radius of the cylinder, representing the size of the exposed subject in the plane orthogonal to the direction of the primary magnetic field). Each curve indicates, as a function of frequency, the maximum extent of the perturbation of the primary field due to the currents induced in the exposed object. To build the curves, the conductivity values as a function of frequency corresponding to human muscle tissue were used, as determined with the parametric model of C. Gabriel [1].

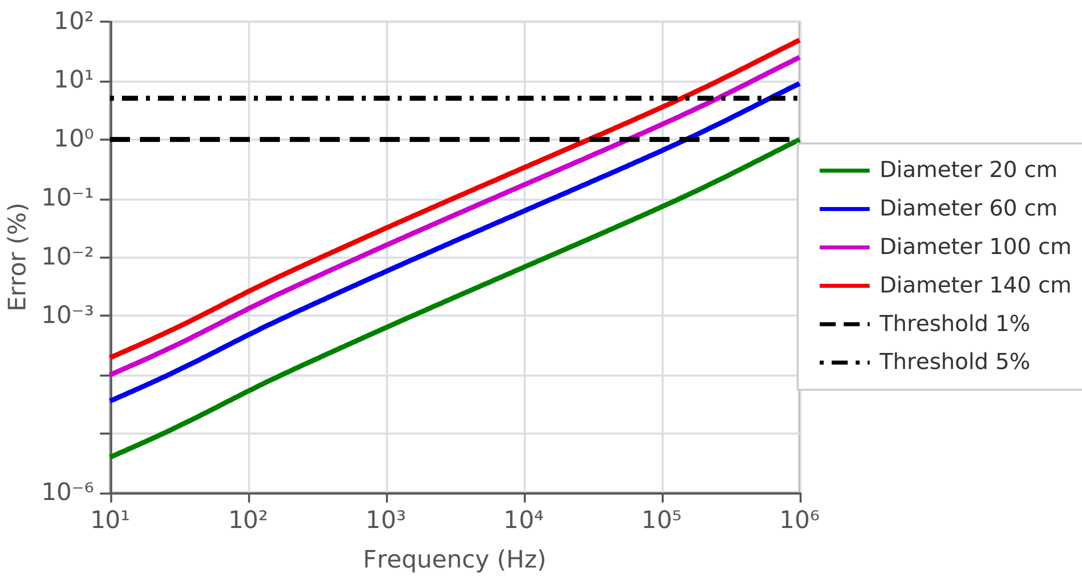

Figure 2. *Maximum extent of the perturbation of the primary magnetic field due to the currents induced in the exposed object, as a function of frequency, for six values of the diameter of the equivalent cylinder. Muscle tissue conductivity after the parametric model of C. Gabriel.*

Appendix

Eq. 5 is obtained from Eq. 4 by means of the following limit operation:

$$\lim_{h\to\infty} h\left(\sqrt{a^2+h^2}-h\right) = \frac{a^2}{2}$$

To prove the validity of this expression, consider that:

$$
\begin{aligned}
h\left(\sqrt{a^2+h^2}-h\right) &= h^2\left(\sqrt{\frac{a^2+h^2}{h^2}}-1\right) = h^2\left(\sqrt{\frac{a^2}{h^2}+1}-1\right) \\
&= h^2\left[\left(1+\frac{a^2}{h^2}\right)^{0.5}-1\right] \approx h^2\left[\left(1+\frac{a^2}{2h^2}\right)-1\right] = \frac{a^2}{2}
\end{aligned}
$$

where the asymptotic relation [2]:

$$
(1+x)^n \approx 1+n\,x \quad \text{if}\, x \ll 1
$$

was used, which is valid in the case at hand (where $x = a^2/h^2$) if $a \ll h$ — a condition certainly met when $h \rightarrow \infty$.

References

[1] D. Andreuccetti, R. Fossi and C. Petrucci: "An Internet resource for the calculation of the dielectric properties of body tissues in the frequency range 10 Hz - 100 GHz". CNR-IFAC, Florence (Italy), 1997. Based on data published by C. Gabriel et al. in 1996. [Online]. Available: http://niremf.ifac.cnr.it/tissprop/

[2] H.B. Dwight: "Tables of integrals and other mathematical data", 4th Edition, The Macmillan Company 1961. Eq. (1) page 1 and Eq. (5.3) page 2.